Generalized Fuzzy Non-Monotonic Reasoning Using Fuzzy Membership Functions Likely and Unlikely

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***Abstract*— Knowledge is key factor. Sometimes knowledge available to the system for reasoning is incomplete. For non-monotonic reasoning, the knowledge available to the systems is incomplete. The bird can fly is not known. Fuzzy logic is able to deal with incomplete information. In this paper, fuzzy non-monotonic logic is studied. Fuzzy non-monotonic reasoning is studied. Fuzzy granular non-monotonic reasoning is studied. Fuzzy truth maintenance system (FTMS) is studied for fuzzy non-monotonic reasoning. Some examples are discussed for fuzzy non-monotonic reasoning**.

Keywords*— *fuzzy Sets, non-monotonic reasoning, fuzzy rezoning, fuzzy non-monotonic reasoning, fuzzy granularity, incomplete knowledge

I. Introduction

Sometimes AI has to deal with incomplete knowledge. If knowledge base is incomplete then inference is also incomplete. If knowledge is added then the inference is changes. Some knowledge is sufficient for reasoning. Sometimes knowledge is not sufficient for complete reasoning. Such situations fall under non-monotonic [10]. There are many theories to deal with incomplete information like Probability, Dempster- Shaffer theory, Possibility, Plausibility, non-monotonic etc. Zadeh [11] fuzzy logic is based on belief rather than probable (likelihood)

In non-monotonic reasoning, some additional information is to be added the reasoning will be changed [4].

"if x is not known then conclude y"

"if x is con not be proved in some amount of time, then conclude y"

x is bird ^ x has wings Λ x is known to fly → x can fly
x is bird ^ x has wings Λ x is not known to fly → x can fly
Ozzie can fly?
It is imprecise.
Ozzie is bird Λ x has wings Λ x is not known to fly → x can't fly

In the above situations, though added some knowledge, it not possible to reasoning with non-monotonic logic.

Zadeh [12] propose Z-Number or Zadeh – Number as Z= (A, B) for the proposition of the type "x is P"

Where A is likely support the knowledge and B is unlikely support the knowledge.

Here is first case support the inference and in second case not support the information.

x is bird Λ x has wings Λ x is likely to fly → x can fly
x is bird ^ x has wings ^ x is unlikely to fly → x can't fly

The fuzzy non-monotonic reasoning will bring imprecise knowledge in to precise knowledge.

In the following, fuzzy non-monotonic logic is discussed for incomplete knowledge of non-monotonic reasoning.

II. Fuzzy Logic

The possibility set may be defined for the proposition of the type "x is P" as

$\pi_P(x) \rightarrow [0,1]$
$\pi_P(x)=\max\{ \mu_P(x_i) \}, x \in X$

$\mu_P(x)= \mu_P(x_1)/x_1+ \mu_P(x_2)/x_2+\ldots+ \mu_P(x_n)/x_n$
$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$
$\mu_{bird}(x)=$ 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos

Let P and Q be the fuzzy sets, and the operations on fuzzy sets are given below [10]

$PVQ=\max(\mu_P(x) , \mu_Q(x)\}$	Disjunction
$P\Lambda Q=\min(\mu_P(x) , \mu_Q(x)\}$	Conjunction
$P'=1- \mu_P(x)$	Negation
$PxQ=\min \{ \mu_P(x) , \mu_Q(x)\}$	Relation
$P \text{ o } Q==\min\{\mu_P(x), \mu_Q(x, x)\}$	Composition

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

$\mu_{very}(x) =\mu_P(x)^2$ Concentration
$\mu_{more\ or\ less}(x) = \mu_P(x)^{0.5}$ Diffusion

The Zadeh [11] fuzzy condition inference s given by

if x is x is P_1 and P_2 …. X is Pn then Q =
min 1, (1-min($\mu_{P1}(x)$, $\mu_{P2}(x)$, …, $\mu_{Pn}(x)$) $+\mu_Q(x)\}$

The Mamdani [5] fuzzy condition inference s given by
if x is x is P_1 and P_2 …. X is Pn then Q =
min {1, (1-min($\mu_{P1}(x)$, $\mu_{P2}(x)$, …, $\mu_{Pn}(x)$) +$\mu_Q(x)$}

The Reddy fuzzy condition inference s given by
if x is x is P_1 and P_2 …. X is Pn then Q =
min($\mu_{P1}(x)$, $\mu_{P2}(x)$, …, $\mu_{Pn}(x)$)

Quasi-fuzzy set

A quasi-fuzzy set is defined for the proposition " x is P" as

$\mu_P(x) \rightarrow (0, 1)$

III. FUUZY NON-MONOTONIC LOGIC

Zadeh [10] is defined the Z-Number {A,B] for the proposition of the type "x is P", where A support the P and B not support the P .

The fuzzy non-monotonic set may defined with two fold membership function using likely and unlikely .

Definition: Given some Universe of discourse X, the proposition " x is P" is defined as its two fold fuzzy membership function as

$\mu_P(x) = \{\mu_P^{likely}(x), \mu_P^{unlikely}(x)\}$
or
$P = \{\mu_P^{likely}(x), \mu_P^{unlikely}(x)\}$
Where P is Generalized fuzzy set and x Є X ,

$0 <= \mu_P^{likely}(x) <=1$ and, $0 <= \mu_P^{unlikely}(x) <=1$
$P = \{ \mu_P^{likely}(x_1)/x_1 + \ldots + \mu_P^{likely}(x_n)/x_n,$
$\mu_P^{unlikely}(x_1)/x_1 + \ldots + \mu_P^{likely}(x_n)/x_n$, x_i Є X, "+" is union

For example ' x will fly" , fly may be given as

fly = {$\mu_{fly}^{likely}(x)$, $\mu_{fly}^{unlikely}(x)$}
={0.1/peacock + .3/hen+0.5/cock+0.6/parrat+0.9/eagle,
0.9/peacock + .8/hen+0.7/cock+0.5/parrat+0.1/eagle]

IV. EXTENTION OF Z-FUZZY LOGIC TO FUZZY NIN-MONOTONIC LOGIC

Since formation of the fuzzy non-monotonic logic is simply two fold fuzzy logic. Zadeh fuzzy Z-fuzzy set is extended to fuzzy non-monotonic logic .
$\mu_P(x) = \{\mu_P^{likely}(x), \mu_P^{unlikely}(x) \}$

Suppose P and Q are fuzzy non-monotonic sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Negation

$P' = \{1- \mu_P^{likely}(x), 1- \mu_P^{unlikely}(x) \}/x$

Disjunction

PVQ={ max($\mu_P^{likely}(x)$, $\mu_P^{likely}(y)$), max($\mu_Q^{unlikely}(x)$, $\mu_Q^{unlikely}(y)$)}(x,y)

Conjunction

PΛQ={ min($\mu_P^{likely}(x)$,$\mu_P^{likely}(y)$), min($\mu_Q^{unlikely}(x)$, $\mu_Q^{unlikely}(y)$) }/(x,y)

Implication

Zadeh fuzzy conditional inference
$P \rightarrow Q$= {min(1, 1- $\mu_P^{likely}(x)$ + $\mu_Q^{likely}(y)$, min (1, 1- $\mu_P^{unlikely}(x)$ + $\mu_Q^{unlikely}(y)$}(x,y)

Mamdani fuzzy conditional inference
$P \rightarrow Q$= {min($\mu_P^{likely}(x)$, $\mu_Q^{likely}(y)$, min ($\mu_P^{unlikely}(x)$, $\mu_Q^{unlikely}(y)$}(x,y)

Reddy fuzzy conditional inference
$P \rightarrow Q$= {min ($\mu_P^{likely}(x)$, $\mu_P^{unlikely}(y)$}(x,x)

Composition

P o R = {$\min_x$ ($\mu_P^{likely}(x)$, $\mu_P^{likely}(x)$), $\min_x$($\mu_R^{unlikely}(x)$, $\mu_R^{unlikely}(x)$)}/y

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

Concentration

"x is very P
$\mu_{very\ P}(x) = \{ \mu_P^{likely}(x)^2, \mu_P^{unlikely}(x)\mu_P(x)^2 \}$

Diffusion

"x is more or less P"

$\mu_{more\ or\ less\ P}(x) = (\mu_P^{likely}(x)^{1/2}, \mu_P^{unlikely}(x)\mu_P(x)^{0.5}$

For instance, consider logical operations on P and Q

P = { $0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 +0.5/x_5$,
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 +0.6/x_5$}

Q = { $0.9/x_1 + 0.7/x_2 + 0.8/x_3$ + 0..5/x_4 +0.6/x_5 ,
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 +0.7/x_5$}

P V Q = { $0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 +0.6/x_5$,
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 +0.7/x_5$}

P Λ Q = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 +0.5/x_5$,
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 +0.6/x_5$}

P′ = not P= { $0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 +0.5/x_5$,
$0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 +0.4/x_5$}

$P \rightarrow$ Q = { $1/x_1 + 0.8/x_2$ + $/x_3$ + $0.9/x_4 +1/x_5$,
$1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 +1/x_5$}
P o Q = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 +0.5/x_5$,
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 +0.6/x_5$}

$\mu_{very\ P}(x) = \{ \mu_P^{likely}(x)^2, \mu_P^{unlikely}(x)\mu_P(x)^2 \}$
= { $0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 +0.25/x_5$,
$0.16/x_1 + 0.09/x_2 + 0.16/x_3 + 0.49/x_4 +0.36/x_5$

$\mu_{\text{more or less P}}(x) = (\mu_L^{ikely} (x)^{1/2}, \mu_U^{nlikely} (x)\mu_P(x)^{1/2} \}$
$=\{ 0.89/x_1 + 0.95/x_2 + 0.84/x_3 + 0.77/x_4 +0.70/x_5 ,$
$0.63/x_1 + 0.55/x_2 + 0.63/x_3 + 0.81/x_4 +0.77/x_5\}$

quasi-fuzzy non-monotonic set is defined as

$\mu_P (x) = \{\mu_P^{likely} (x), \mu_P^{unlikely} (x)\}$
$\mu_P (x) \rightarrow (0, 1)$

Consider the fuzzy non-monotonic inference

"x is bird Λ x is known to fly then x can fly"

$\mu_{bird}(x) = \{\mu_{bird}^{likely} (x), \mu_{bird}^{unlikely} (x)\}$

$\mu_{bird}(x)=$ { 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos , 0.9/Penguin + 0.7/Hen+0.6/Cock + 0.4/Parrot + 0.2/eagle + 0.0/flamingos }

$\mu_{known} (x)=1$

$\mu_{bird}(x)=$ { 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos , 0.9/Penguin + 0.7/Hen+0.6/Cock + 0.4/Parrot + 0.2/eagle + 0.0/flamingos }

"x is bird Λ x is known to fly then x can fly" is gen by

$\mu_{fly}(x)= \mu_{bird}(x) \wedge \mu_{known}(x)=$
$\mu_{fly}(x)=$ { 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos , 0.9/Penguin + 0.7/Hen+0.6/Cock + 0.4/Parrot + 0.2/eagle + 0.0/flamingos } Λ { 1.0/Penguin + 01.0/Hen+1.0/Cock + 1.0/Parrot + 01.0/eagle + 1.0/flamingos , 1.0/Penguin + 1.0/Hen+1.0/Cock + 1.0/Parrot + 01.0/eagle + 1.0/flamingos }

= { 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos , 0.9/Penguin + 0.7/Hen+0.6/Cock + 0.4/Parrot + 0.2/eagle + 0.0/flamingos }

"x is bird Λ x is not known to fly then x can fly" is gen by

$\mu_{fly}(x)= \mu_{bird}(x) \Lambda \mu_{\text{not known}}(x) =$
{ 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos , 0.9/Penguin + 0.7/Hen+0.6/Cock + 0.4/Parrot + 0.2/eagle + 0.0/flamingos } ^ { 0.0/Penguin + 0.0/Hen+0.0/Cock + 0.0/Parrot + 0.0/eagle + 0.0/flamingos , 0.0/Penguin + 0.0/Hen+0.0/Cock + 0.0/Parrot + 0.0/eagle + 0.0/flamingos }
= { 0.0/Penguin + 0.0/Hen+0.0/Cock + 0.0/Parrot + 0.0/eagle + 0.0/flamingos , 0.0/Penguin + 0.0/Hen+0.0/Cock + 0.0/Parrot + 0.0/eagle + 0.0/flamingos }

V. FUZZY GRANULAR NON-MOTONIC LOGIC REASONING

Zadeh[13] defined fuzzy granularity for the proposition of type " x is A is λ" where λ is granular variable likely, unlikely, very likely not very likely, more or less likely, etc.

For instance, the inference for "x is bird is not very likely " is given as
$1- \mu_{bird}(x)$ 2

Fuzzy granular non-monotonic position "x is P is not very likely " is given by
$\{ 1-\mu_P^{likely} (x)^2 , 1-\mu_P^{unlikely}(x)^2\}$

The g fuzzy granular non-monotonic position "x is P is not very unlikely " is given by

$\{ \mu_P^{likely} (x) , \mu_P^{unlikely} (x)\}$

Granular variables may be apply on respective functions
For instance "x is young is likely " is given as
$P =\{ \mu_{young}^{likely} (x) , \mu_{young}^{unlikely} (x)\}$

The fuzzy granular vales may be applied on respective fuzzy membership functions.
"x is P is very likely " is given as

$\{ \mu_{\text{very P}}^{likely} (x) , \mu_P^{unlikely} (x)\}$
"x is P is more or less unlikely " is given as

$\{ \mu_P^{likely}(x) , \mu_{\text{more or less P}}^{unlikely}(x)\}$
For instance, "Ozzie is bird is very likely " is given as

$\{ \mu_{bird}^{likely}$ (Ozzie) , $\mu_{bid}^{unlikely}$ (Ozzie)}

"Ozzie is bird is more or less unlikely " is given as
P{ μ_{bird}^{likely} (Ozzie) , $\mu_{bird}^{unlikely}$ (Ozzie)$^{0.5}$}

VI. FUZZY TRUTH MAITANACE SYSTEM

In the truth maintenance system (TMS) for proposition is give by

x is bird Λ x has wings ^ x is known to fly → x can fly
1. x is known to fly
2. x is not known to fly
3. x can fly
4. x can't fly

IN=input of belief
OUT=output of belief

IN	OUT
x is known to fly	x can fly
x is not known to fly	x can fly
x is not known to fly	x can fly

For instance, Ozzie is bird

IN	OUT
Ozzie is known to fly	Ozzie can fly
Ozzie is not known to fly	Ozzie can't fly
Ozzie is not known to fly	Ozzie can fly

The fuzzy truth maintenance systems (FTMS) will bring the imprecise proposition in to precise proposition.

In the fuzzy truth maintenance system (FTMS) for proposition is give by

x is bird Λ x has wings Λ x is known to fly → x can fly

1. x is known to fly
2. x can fly
3. x can't fly

IN=input of likely, unlikely
OUT=output is belief

IN	OUT
x is no likely to fly	x can fly
x is known unlikely to fly	x can't fly

For instance, Ozzie is bird

IN OUT
IN OUT

Ozzie is known likely to fly	Ozzie can fly
Ozzie is known unlikely to fly	x can't fly

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