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The Hybrid Chain Corrections for ZZ and γγ Pair Production in the SM at Future e^+e^- Colliders

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The hybrid chain corrections for ZZ and $\gamma\gamma$ pair production in the SM at future $\mathrm{e^+e^-}$ colliders

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***Abstract*—We present the precision predictions for neutral gauge boson pair production in the standard model (SM) at the future $\mathrm{e^+e^-}$ colliders including the electron hybrid chain electroweak correction. The differential cross section of $\mathrm{e^+e^-} \to \mathrm{ZZ}, \gamma\gamma$ by the renormalized electron hybrid chain propagator (RHCP) is calculated analytically. Besides, the differential cross section between cases of the RHCP and Born level are compared. The numerical results show that, the relative correction of RHCP with respect to tree level for $\mathrm{e^+e^-} \to \mathrm{ZZ}$ are located in a rannge mainly from 2.52% to 3.41% in the range of the center-of-mass energy from 184GeV to 300GeV, and the radiative corrections increases with the increase of incident electron energy for $\mathrm{e^+e^-} \to \gamma\gamma$ in the incident energy from 1GeV to 200GeV.**

***Index Terms*—Standard Model; Hybrid chain propagator; Cross section; Relative correction.**

I. Introduction

The Standard Model of electroweak interactions is the most influential phenomenological theory in particle physics. It has made great success in describing the weak and electromagnetic interactions, and the new physics predicted by SM have been confirmed in many experiments, especially the Higgs particle was founded in July 2012 by both the ATLAS and the CMS collaborations at LHC[1][2]. Since the standard model has been proposed by Weinberg and Salam, it has aroused a serious of related theoretical studiesłand the works involving the "precision" test of SM the theoretical earned special academic concern all along[3][4][5][6]. These theoretical s-tudies have proposed the development of SM. Moreover, using perturbative quantum field theory to do calculation is the most important works when we do theoretical calculations.

Theoretical predictions should have an accuracy compared to or even better than the experimental errors. So we are forced to take into account high order corrections if we want to do accurate calculations. Usually some intricate intermediate states and divergent troubles[7][8][9] will be mentioned. Therefore, it is necessary to employ the renormalization techniques (e.g. often used Feynman integrals[10] and other realizations[11][12][13][14]) to eliminate the unphysical divergent quantity whilst to retain the renormalized physical finite quantity. However, these high-order radiative corrections would still be small even if they can be given by various renormalization schemes. Notwithstanding, their subtle contributions actually present the significance for in-depth exploring the physical questions with advanced precision[15][16][17][18][19]. The contribution of renormalized finite quantity (radiation correction) is very smallbut this small contribution is very important to the deep study of related physical problems[20][21][22][23]. If we can acquire the exact result of this tiny correction effectively, this is obviously can reflect radiation correction of physical matter more accurately. Furthermore, the exact results are also helpful to the study and discussion of physical problems in depth. Although these high-order calculations are normally quite difficult and much less applied, still, in some situations, we can also use certain feasible methods to deal with the high-order, and even infinite high-order corrections. For example, the infinite order chain propagator (its structure covers the one loop diagrams, two loops diagrams ..., until infinite order diagrams in the manner of chains) proposed by Dyson[24][25] and later in detail addressed by Lurie[26] and others, could supply an effective way to contribute finite results of the corrections.

In this paper, we study the hybrid chain corrections for ZZ and $\gamma\gamma$ pair production in the SM at future $\mathrm{e^+e^-}$ colliders, such as The Circular Electron-Positron Collider (CEPC) and International Linear Collider (ILC). We calculate the differential cross section of $\mathrm{e^+e^-} \to \mathrm{ZZ}, \gamma\gamma$ by RHCP analytically. The information about radiative corrections can be obtained by comparing this chain differential cross section with the Born level, and the numerical results show that the relative corrections mainly in a scope from 2.52% to 3.41% for $\mathrm{e^+e^-} \to \mathrm{ZZ}$ and 0.14% to 3.14% for $\mathrm{e^+e^-} \to \gamma\gamma$ in typical parameter range. These outcomes may not only indicate that the theoretical analysis and the calculations are valid, but also support that the considering of hybrid chain propagators and corresponding radiative corrections would be valuable.

The rest of the paper is organized as follows. Section II gives the results of differential cross sections of $\mathrm{e^+e^-} \to \mathrm{ZZ}, \gamma\gamma$ by huybrid chain propagators. In Section III, the numerical results of the relative corrections of renormalized hybrid chain propagators with respect to tree level are carried out. Finally, we conclude the work in Section IV.

II. High Order Corrections of $\mathrm{e^+e^-} \to \mathrm{ZZ}, \gamma\gamma$ Differential Cross Section by RHCP

The Feynman diagrams of $\mathrm{e^+e^-} \to \mathrm{ZZ}, \gamma\gamma$ at tree level and hybrid chain propagator corrections level are shown in Fig 1.

We discuss the process in the center-of-mass system. The explicit expressions for the four-momenta are $p_{1,2}$ =

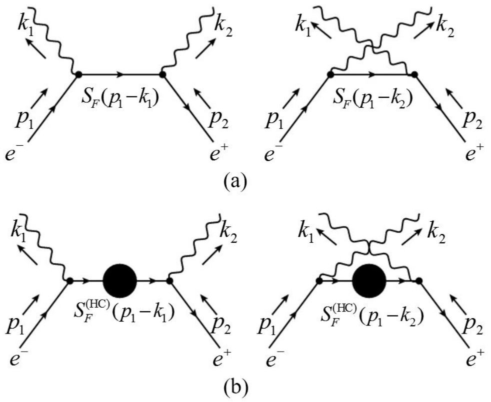

Fig. 1. The Feynman diagrams of $e^+e^- \to ZZ, \gamma\gamma$: (a) tree level; (b) hybrid chain level.

$(E, 0, 0, \mp\sqrt{E^2 - m_e^2})$ and $k_{1,2} = (E, \mp k \sin\theta, 0, \mp k \cos\theta)$. The Mandelstam variables are defined as usual

$$\begin{cases} s = (p_1 + p_2)^2 = (k_1 + k_2)^2 = 4E^2 \\ t = (p_1 - k_1)^2 = (p_2 - k_2)^2 \\ u = (p_1 - k_2)^2 = (p_2 - k_1)^2 \end{cases} \tag{1}$$

Specifically,

$$t = m_Z^2 - \frac{s}{2} + \frac{1}{2}\sqrt{s(s - 4m_Z^2)}\cos\theta \tag{2}$$

$$u = m_Z^2 - \frac{s}{2} - \frac{1}{2}\sqrt{s(s - 4m_Z^2)}\cos\theta \tag{3}$$

In line with the quantum field theory[27], the differential cross section and scattering amplitude of the scattering process $e^+e^- \to ZZ$ obey the relationship

$$\frac{d\sigma_{ZZ}}{d\cos\theta} = \frac{1}{32\pi s}\sqrt{1 - \frac{4m_Z^2}{s}} \cdot \frac{1}{4}\sum_{\text{spin}} |M(p_1, p_2 \to k_1, k_2)|^2 \tag{4}$$

Where, s is the center-of-mass energy, θ is the scattering angle; $\frac{1}{4}\sum\limits_{\text{spin}} |M|^2$ is the squared amplitude, averaged and summed over spins.

For the scattering process $e^+e^- \to ZZ$, the tree level and hybrid chain level scattering amplitude is given by Eq.(3) and Eq.(4), respectively.

$$\begin{aligned} iM_0 = &\bar{\nu}(p_2) ie\gamma^\mu s_c \varepsilon_\mu^*(k_2) S_F(p_1 - k_1) ie\gamma^\nu s_c \varepsilon_\nu^*(k_1) u(p_1) \\ &+ \bar{\nu}(p_2) ie\gamma^\nu s_c \varepsilon_\nu^*(k_1) S_F(p_1 - k_2) ie\gamma^\mu s_c \varepsilon_\mu^*(k_2) u(p_1) \end{aligned} \tag{5}$$

$$\begin{aligned} iM_C = &\bar{\nu}(p_2) ie\gamma^\mu s_c \varepsilon_\mu^*(k_2) S_F^{(HC)}(p_1 - k_1) ie\gamma^\nu s_c \varepsilon_\nu^*(k_1) u(p_1) \\ &+ \bar{\nu}(p_2) ie\gamma^\nu s_c \varepsilon_\nu^*(k_1) S_F^{(HC)}(p_1 - k_2) ie\gamma^\mu s_c \varepsilon_\mu^*(k_2) u(p_1) \end{aligned} \tag{6}$$

Where, $s_c = \frac{1}{4 s_W c_W}(1 - 4s_W^2 - \gamma_5)$, $S_F(p) = \frac{1}{p^2 - m_e^2}(i\gamma \cdot p + m_e)$ is the electron tree propagators, $S_F^{(HC)}(p_1 - k_1)$ and $S_F^{(HC)}(p_1 - k_2)$ are the hybrid chain propagators of electron.

The electron chain propagator also can be expressed as after introducing four corrective parameters[28]

$$\begin{aligned} S_F^{(HC)}(p) = &\frac{i\hat{p}}{p^2 - m_e^2}\xi_1(p) + \frac{im_e}{p^2 - m_e^2}\xi_2(p) \\ &+ \frac{i\hat{p}\gamma_5}{p^2 - m_e^2}\xi_3(p) + \frac{i\gamma_5}{p^2 - m_e^2}\xi_4(p) \end{aligned} \tag{7}$$

Compared to the electron tree propagator $S_F(p) = \frac{i\hat{p}}{p^2 - m_e^2} + \frac{m_e}{p^2 - m_e^2}$, the $\xi_{i=1,2,3,4}(p)$ in formula (7) can be interpreted as corrective parameters. Table I and Table II give the corrective parameters $\xi_1(p), \xi_2(p), \xi_3(p), \xi_4(p)$ for p^2.

TABLE I
CORRECTIVE PARAMETERS $\xi_1(p), \xi_2(p)$ FOR p^2

$p^2(\text{GeV}^2)$	$\xi_1(p)$	$\xi_2(p)$
-1000^2	$0.9890 - 0.0100i$	$0.9417 - 0.0033i$
-100^2	$0.9924 - 0.0101i$	$0.9531 - 0.0032i$
-10^2	$0.9952 - 0.0102i$	$0.9636 - 0.0030i$
-1	$0.9978 - 0.0102i$	$0.9741 - 0.0029i$
1	$0.9979 - 0.0070i$	$0.9740 - 0.0019i$
10^2	$0.9952 - 0.0069i$	$0.9635 - 0.0018i$
100^2	$0.9929 - 0.0071i$	$0.9534 - 0.0016i$
1000^2	$0.9890 - 0.0100i$	$0.9417 - 0.0005i$

It can be seen from table I that the relative correction of $\xi_1(p)$ and $\xi_2(p)$ is not more than 5%, which is accord with the magnitude of electroweak correction. It also can be seen from table II that $\xi_3(p)$ and $\xi_4(p)$ are very small, which means that although there are two extra terms ($\xi_3(p)$ and $\xi_4(p)$) in electron chain propagator compared to the tree propagator, their contributions are very small.

TABLE II
CORRECTIVE PARAMETERS $\xi_3(p), \xi_4(p)$ FOR p^2

$p^2(\text{GeV}^2)$	$10^2 \times \xi_3(p)$	$10^5 \times \xi_4(p)$
-1000^2	$-1.1541 + 0.4278i$	$1.8786 - 0.4996i$
-100^2	$-1.3518 + 0.4347i$	$1.3659 - 0.4911i$
-10^2	$-1.3821 + 0.4377i$	$0.9449 - 0.4822i$
-1	$-1.3897 + 0.4401i$	$0.5307 - 0.4729i$
1	$-1.3978 + 0.5756i$	$0.5415 - 0.2352i$
10^2	$-1.3908 + 0.5721i$	$0.9555 - 0.2399i$
100^2	$-1.4389 + 0.8304i$	$1.3403 - 0.2501i$
1000^2	$-1.1651 + 0.2606i$	$1.8776 - 0.3968i$

After a serious calculation, we have

$$\frac{1}{4}\sum_{\text{spin}} |M_C|^2 = \frac{e^4}{32 s_W^4 c_W^4}\left(\frac{\alpha}{t^2} + \frac{\beta}{tu} + \frac{\kappa}{u^2}\right) \tag{8}$$

With

$$\begin{aligned} \alpha = &\left(1 + 6c_V^2 + c_V^4\right)\left(|\xi_1(t)|^2 + |\xi_3(t)|^2\right)(tu - m_Z^2 s) \\ &- 8\left(c_V + c_V^3\right)|\xi_1(t)\xi_3(t)^*|(tu - m_Z^2 s) \end{aligned} \tag{9}$$

$$\begin{aligned} \beta = &\left[\left(1 + 6c_V^2 + c_V^4\right)\left(|\xi_1(t)\xi_1(u)^*| + |\xi_3(t)\xi_3(u)^*|\right)\right. \\ &\left. + 8\left(c_V + c_V^3\right)|\xi_1(t)\xi_3(u)^*|\right]\left(2m_Z^2 s + t^2 + u^2 - s^2\right) \end{aligned} \tag{10}$$

$$\begin{aligned} \kappa = &\left(1 + 6c_V^2 + c_V^4\right)\left(|\xi_1(u)|^2 + |\xi_3(u)|^2\right)(tu - m_Z^2 s) \\ &- 8\left(c_V + c_V^3\right)|\xi_1(u)\xi_3(u)^*|(tu - m_Z^2 s) \end{aligned} \tag{11}$$

We can directly acquire the tree level result by setting $\xi_1 = \xi_2 = 1$ and $\xi_3 = \xi_4 = 0$. The analytical result of tree level can be written as

$$\frac{d\sigma^0_{ZZ}}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\frac{1{+}6c_{\rm V}^2+c_{\rm V}^4}{32s_{\rm W}^4c_{\rm W}^4}\sqrt{1-\frac{4m_Z^2}{s}}\left(\frac{tu-m_Z^2 s}{t^2} + \frac{tu-m_Z^2 s}{u^2} + \frac{2m_Z^2 s{+}t^2{+}u^2-s^2}{tu}\right) \tag{12}$$

For the scattering process $\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma$, the tree level and hybrid chain level scattering amplitude is given by

$$\begin{aligned} iM_0 = &\bar{\nu}(p_2)ie\gamma^\mu\varepsilon^*_\mu(k_2)S_F(p_1-k_1)ie\gamma^\nu\varepsilon^*_\nu(k_1)u(p_1) \\ &+\bar{\nu}(p_2)ie\gamma^\nu\varepsilon^*_\nu(k_1)S_F(p_1-k_2)ie\gamma^\mu\varepsilon^*_\mu(k_2)u(p_1) \end{aligned} \tag{13}$$

$$\begin{aligned} iM_{\rm C} = &\bar{\nu}(p_2)ie\gamma^\mu\varepsilon^*_\mu(k_2)S_F^{\rm (HC)}(p_1-k_1)ie\gamma^\nu\varepsilon^*_\nu(k_1)u(p_1) \\ &+\bar{\nu}(p_2)ie\gamma^\nu\varepsilon^*_\nu(k_1)S_F^{\rm (HC)}(p_1-k_2)ie\gamma^\mu\varepsilon^*_\mu(k_2)u(p_1) \end{aligned} \tag{14}$$

Similar to $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ}$, the differential cross section for the tree level is given by

$$\frac{d\sigma^0}{d\cos\theta} = \frac{2\pi\alpha^2}{s}\frac{E}{\sqrt{E^2-m_e^2}}\left[\frac{E^2+p^2\cos\theta^2}{m_e^2+p^2\sin\theta^2} + \frac{2m^2}{m^2+p^2\sin\theta^2} - \frac{2m^4}{(m^2+p^2\sin\theta^2)^2}\right] \tag{15}$$

The differential cross section also can be written as Eq.(14) if we ignore the mass of electron.

$$\frac{d\sigma^0_{\gamma\gamma}}{d\cos\theta} = \frac{2\pi\alpha^2}{s}\frac{(1+\cos\theta^2)}{\sin\theta^2} \tag{16}$$

Similarly , the differential cross section for the hybrid chain level is given by

$$\frac{d\sigma^{\rm (HC)}_{\gamma\gamma}}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[\left(|\xi_1(t)|^2+|\xi_3(t)|^2\right)\left(\frac{1+\cos\theta}{\sin\theta}\right)^2 + \left(|\xi_1(u)|^2+|\xi_3(u)|^2\right)\left(\frac{1-\cos\theta}{\sin\theta}\right)^2\right] \tag{17}$$

We have also neglected the electron mass whenever possible during the process, so in this case the Mandelstam variables $u = -s(1+\cos\theta)/2$, and $t = -s(1-\cos\theta)/2$. The analytical result of tree level can also be directly acquired by setting $\xi_1 = 1$ and $\xi_3 = 0$.

III. Radiative Corrections Contributed by the High-Order Hybrid Chain Propagators

Table III and Table IV give the radiative corrections caused by infinite order RHCPs in the process of $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ}$ and $\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma$, against the scattering angle and the center of energy $E_{\rm cm}$(GeV) in typical ranges. The value of radiative correction here is defined as $\left|\frac{d\sigma^{\rm (HC)}}{d\cos\theta} - \frac{d\sigma^0}{d\cos\theta}\right| \Big/ \frac{d\sigma^0}{d\cos\theta}$. We can find noticeable radiative corrections in Table III, and their values are mainly less than 0.035 (mostly around 2.52% to 3.41%) in a typical range of the incident energy from 184GeV to 300GeV. It can be also found that the radiative corrections increases with the increase of incident electron energy for $\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma$ in Table IV. Intuitive representations of the relationships between the differential cross section and the scattering angle , with comparisons between cases of the RHCP and the tree-level propagator, are revealed in Fig.2 and Fig.3, given different values of the incident energy or scattering angle, and these distinctions shown no doubt reflect differences between the differential cross sections with RHCPs and the differential cross sections with only tree propagators. All these results play some supporting roles to argue that the considering of the RHCPs would be valuable, and the obtained high-order radiative corrections are effective for providing further enhanced precision.

TABLE III
THE RADIATIVE CORRECTIONS CAUSED BY INFINITE ORDER RHCPS IN THE PROCESS OF $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ}$, AGAINST THE SCATTERING ANGLE θ AND THE CENTER OF ENERGY $E_{\rm CM}$ (GeV) IN TYPICAL RANGES

$E_{\rm cm}/\theta$	0.1π 0.9π	0.2π 0.8π	0.3π 0.7π	0.4π 0.6π	0.5π
184GeV	2.81%	2.82%	2.83%	2.83%	2.84%
190GeV	2.75%	2.79%	2.83%	2.86%	2.86%
200GeV	2.67%	2.76%	2.86%	2.91%	2.94%
220GeV	2.58%	2.77%	2.92%	3.01%	3.03%
240GeV	2.53%	2.80%	3.00%	3.09%	3.11%
250GeV	2.52%	2.82%	3.04%	3.13%	3.41%
300GeV	2.68%	2.99%	3.21%	3.30%	3.29%

TABLE IV
THE RADIATIVE CORRECTIONS CAUSED BY INFINITE ORDER RHCPS IN THE PROCESS OF $\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma$, AGAINST THE SCATTERING ANGLE θ AND THE CENTER OF ENERGY $E_{\rm CM}$ (GeV) IN TYPICAL RANGES

$E_{\rm cm}/\theta$	0.1π 0.9π	0.2π 0.8π	0.3π 0.7π	0.4π 0.6π	0.5π
1GeV	0.14%	0.18%	0.37%	0.51%	0.56%
10GeV	0.92%	1.23%	1.39%	1.54%	1.62%
50GeV	1.69%	2.15%	2.41%	2.59%	2.67%
100GeV	2.15%	2.59%	2.85%	3.03%	3.10%
200GeV	2.53%	2.81%	3.03%	3.12%	3.14%

IV. Conclusions

In this paper, we analyzed and discussed the construction of electron propagator and its renormalization in detail via SM, and obtained the analytical result of it. The renormalization model of this paper, not only taking into the part infinite high order situation of perturbation theory, but the renormalized constants of counter terms into physical parameter reasonably. By employing the RHCP, through lengthy calculations such as using the trace technology, the unpolarized differential cross section of $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ}, \gamma\gamma$ is worked out. Via comparisons of unpolarized differential cross sections between cases of the RHCP and the tree propagator, clear radiative corrections are presented.

The results acquired in this paper would argue the following key points: (i) The infinite high-order calculations rather than the low order treatments can be applied for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ}, \gamma\gamma$ under some particular situation; (ii) The effect of infinite order radiative correction can be analytically expressed in a finite and strict manner by use of relevant series summation; (iii) The radiative corrections obtained here are obvious and located in

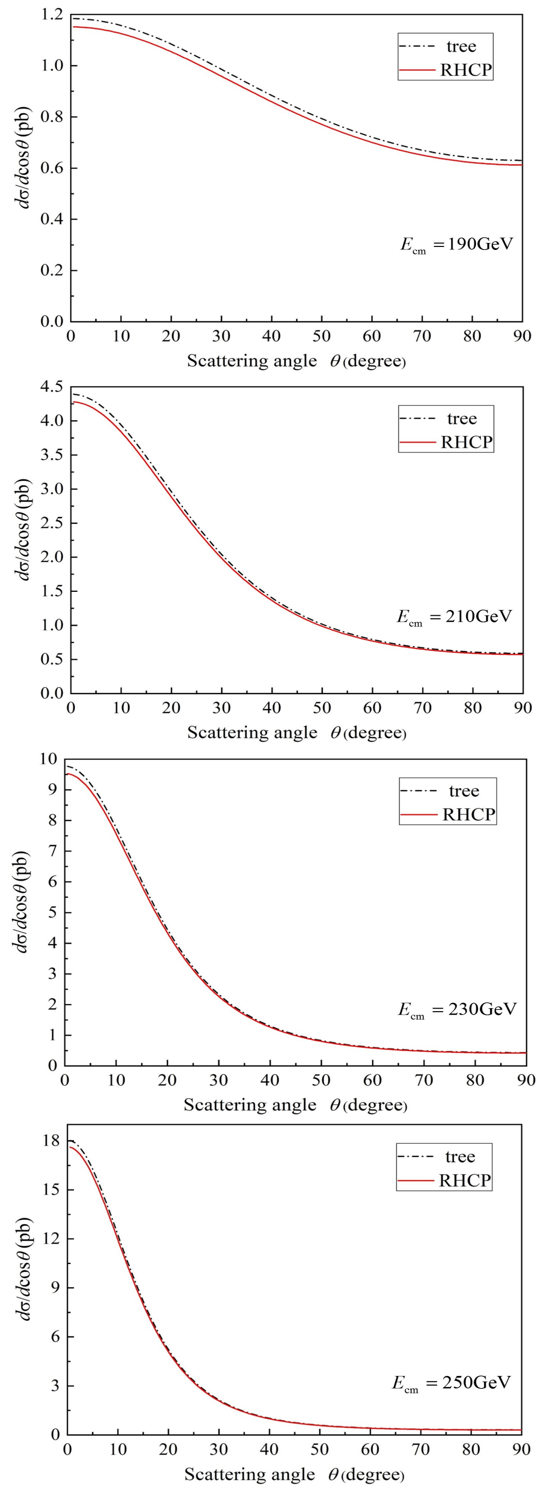

Fig. 2. Comparison of unpolarized differential cross section of $e^+e^- \to ZZ$, between cases of the infinite order RHCP and the tree propagator, given typical scattering angles θ.

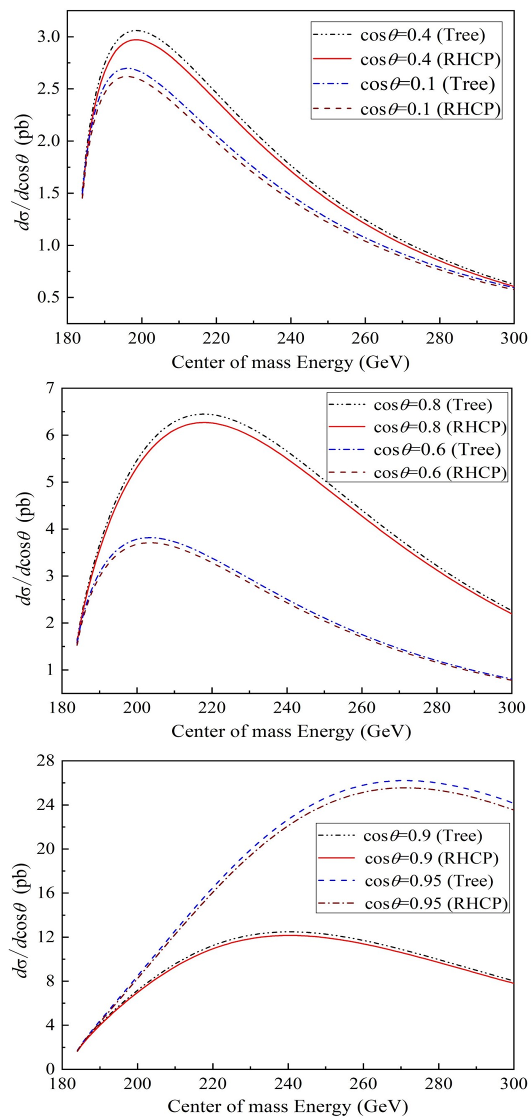

Fig. 3. Comparison of unpolarized DCSs of $e^+e^- \to ZZ$, between cases of the infinite order RHCP and the tree propagator, given typical values of the centre-of-mass energy 190GeV to 300GeV.

a range mainly from 2.52% to 3.41% for $e^+e^- \to ZZ$ in a typical range of the incident energy from 184GeV to 300GeV; (iv) The radiative corrections increases with the increase of incident electron energy for $e^+e^- \to \gamma\gamma$ in the incident energy from 1GeV to 200GeV; (v) The analytical DCS and radiative corrections achieved in this paper bring us some cogent affirmation of the meaning and novelty to consider the RHCPs. The outcomes would increase our understanding of additional approaches towards some infinite order radiative corrections. As one of the most challenging and compelling topics in the particle physics, the high-order corrections and renormalization schemes never stop moving forward, from Dyson[25] to some later contributors[29][30][31]. While, most of existing research works only handle the calculations with finite orders, because in the context of phenomenological theory, the high-order procedure still leaves lots of tough problems unanswered and many avenues of inquiry unexplored. Whereas, the renormalization method implemented in this paper based on the RHCP not only takes the infinite order into consideration, but also suggests an effective approach to acquire the convergent radiative corrections. Further, except this specific sort of the RHCP focused here, it does not exclude other diverse propagators with more complicated structures and various components (e.g. the photon chain propagators with renormalized single photon loop).

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