

Neural Network Modeling as a Method for Creating Digital Twins: From Industry 4.0 to Industry 4.1

Alexandra Dashkina, Ludmila Khalyapina, Aleksandra Kobicheva, Tatiana Lazovskaya, Galina Malykhina and Dmitriy Tarkhov

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Dashkina Alexandra Peter the Great St. Petersburg Polytechnic University Saint-Petersburg, Russia wildroverprodigy@yandex. ru

Lazovskava Tatiana Computing Center of the Far Eastern Branch of the Russian Academy of Sciences Khabarovsk, Russia tatianala@list.ru

Khalyapina Ludmila Peter the Great St. Petersburg Polytechnic University Saint-Petersburg, Russia lhalapina@bk.ru

Malykhina Galina St. Petersburg Polytechnic University of Peter the Great, St. Petersburg, Russia g f malychina@mail.ru

Kobicheva Aleksandra Peter the Great St. Petersburg Polytechnic University Saint-Petersburg, Russia kobicheva92@gmail.com

Tarkhov Dmitriy Peter the Great St. Petersburg Polytechnic University Saint-Petersburg, Russia dtarkhov@gmail.com

ABSTRACT

Digital twins are one of the key technologies behind the Fourth Industrial Revolution. In the coming years they will be introduced on a large scale in the industry and in other spheres. A wide range of digital twins will be in demand: from separate components to complex technical facilities, such as automobiles, airplanes, manufacturing lines, factories, corporations, etc. To provide their successful interaction, it is important to create digital twins on the uniform principles. Currently, creating a digital twin is a complex scientific issue. It presents difficulties because it is necessary not only to describe physical (or chemical, biological, etc.) processes going on in the object, but also to envisage significant changes of its properties in the course of its operation. In this case the digital twin is supposed to adapt to the changes in the original object in accordance with the data received from the sensors.

When the real object is in operation, its properties and specifics of the physical processes going on in it can change. The model is supposed to adapt in accordance with these changes, which is rather difficult if a model is generated by applying computer-aided engineering software packages (CAE) based on the finite element method (FEM).

We think that another approach is more promising. It involves building an adaptive model at the second stage. This model can be specified and redesigned in accordance with the observations on the object. Since neural networks have proved to be efficient in solving complicated problems related to data processing, we recommend using them as the basic class of mathematical models for creating digital twins.

CCS CONCEPTS

• Applied computing \rightarrow Economics • Social and professional topics \rightarrow Economic impact

KEYWORDS

Industry 4.0, digital twins, mathematical model, data set, neural network modelling.

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1. INTRODUCTION

Industry 4.0 is largely based on the digital twin technology. Digital twins corresponding to particular physical objects will work in computational nodes that control manufacturing lines, robots, complex technical installations (airplanes, automobiles, vessels, etc.) Digital twins appear to be the most useful when the properties of the object change over its lifetime. These changes can be undesirable, for instance, wear and tear of friction surfaces or the formation of cracks in metal components. In [1] it is suggested that it should be used for monitoring supply chains, which will both lead to improvements in the manufacturing process and provide feedback with the supplier of raw materials. In [2] the authors consider a number of spheres in which digital twins can be applied: smart homes, hotels and hospitals; smart cities, classes and campuses, smart stores, warehouses and production floors; virtual and augmented reality, smart and safe hospitals. Digital twins for autonomous / self-driving cars can make decisions about planning the route and interacting with other vehicles.

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Currently, digital twins have been defined in a number of ways by various authors. For example, the Defense Acquisition University defined a digital twin in the following way: "an integrated multiphysics, multiscale, probabilistic simulation of an as - built system, enabled by Digital Thread, that uses the best available models, sensor information, and input data to mirror and predict activities/performance over the life of its corresponding physical twin". In the article [1] a digital twin is defined as:

- A model of the object;
- An evolving set of data relating to the object, and

- A means of dynamically updating or adjusting the model in accordance with the data.

In the article [2] the authors give the following definition of a digital twin: " a digital twin is a computer program that takes real-world data about a physical object or system as inputs and produces as outputs (predications or simulations of how that physical object or system will be affected by those inputs)".

In the article [3]"DT is regarded consistently as a high fidelity virtual replica of the physical asset with real-time two-. way communication for simulation purposes and decision-making aiding features for product service enhancement". Söderberg with a group of researchers [4] define a digital twin as "Using a digital copy of the physical system to perform real-time optimization". According to Kannan and Arunachalam [5] a digital twin is a "Digital representation of the physical asset which can communicate, coordinate and cooperate the manufacturing process for an improved productivity and efficiency through knowledge sharing".

Problems: Once digital twins are embedded in a computational node, they cannot be kept unchanged since the simulated object undergoes changes in the course of functioning. A virtual twin of the real object should change in accordance with the information received from the sensors. The algorithms of such changes should be implemented in the above mentioned computational node. For example, it is clear that a unified DT modeling framework is needed urgently [3]. In order to make a digital model of a physical object it is necessary to obtain information about its geometry and the properties of the material it is made from. One reason that the digital twin concept is so valuable in manufacturing is that it allows for development of individual models of individual objects within a unifed framework that makes model development, validation, and updating simple. An individually tailored model can be used for many applications during manufacture and service [1].

When complex technical facilities are designed, it is common to apply computer-aided engineering software packages (CAE) based on the finite element method (FEM) - ANSYS, ABAQUS, etc. However, using them for modelling a real object poses a number of major difficulties. Firstly, to apply FEM it is necessary to know differential equations describing the behavior of the object. Adequate information about these equations is not usually available because it is difficult to describe physical processes going on in the object which is being emulated. Secondly, applying FEM requires knowledge of initial and boundary conditions, but this information is generally even less accurate and adequate. Thirdly, when the real object is in operation, its properties and specifics of the physical processes going on in it can change. The model is supposed to adapt in accordance with these changes, which is rather difficult if a model is generated by FEM.

In the article [6] it is argued that one of the priorities is developing a unified approach and a consolidated definition of a digital twin. It applies primarily to transition from niche, intra-industry projects to a general one, in which all implementations of digital twins will be compatible and capable of integration.

Recently a number of approaches to creating digital twins have been used. Emuakpor et al. integrated a nondestructive material determination technique, a water displacement method, and an iterative Ritz method for the DT to measure the material property. The technique was verified through an experiment on nickel alloys [7]. [«Digital Twin in Industry: State-of-the-Art» Fei Tao, Senior Member, IEEE, He Zhang, Ang Liu, and A.Y.C. Nee]. Majumdar et al. studied the behavior of synergistic materials based on the multi-physics modeling, which was used as the foundation for building the DT model [8]. [«Digital Twin in Industry: State-of-the-Art» Fei Tao, Senior Member, IEEE, He Zhang, Ang Liu, and A.Y.C. Nee].

5G technologies [6] have played an increasingly significant part in creating digital twins since their development has opened up more opportunities for faster providing and exchanging data.

In the course of transition to Industry 4.0 it is essential to create appropriate mathematical and algorithmic tools, which will be conducive to addressing a wide range of issues in a uniform and consistent way. It will be the next significant milestone for transition to Industry 4.1, which will involve the same production processes as Industry 4.0, but the implementation of these processes will be achieved on the basis of cheaper unified technologies.

2. METHODS

We consider neural networks to be one of such key technologies. We suggest that neural networks should be applied for creating digital twins. [9],[10],[11].

Currently neural networks are widely used in Big Data problems, image processing, pattern recognition, controlling complex systems and other artificial intelligence issues. They can be applied to some of the problems related to creating digital twins. For example, in article [12] a general model of ignition processes and forest fire propagation was developed. This model was built on the same principles as digital twins. The general model includes several "submodels", which form a hierarchy. A discrete empirical "submodels" tracks the trajectory of airborne hot particles. A topographical "submodel" takes into consideration the ambient combustible material, airborne embers capable of starting secondary fires as well as updrafts (due to hot air). The third "submodel", Connectionist Modeling as a Method for Creating Digital Twins: From Industry 4.0 to Industry 4.1

based on machine learning allows considering data received from sensors and cameras.

Each "submodel" is supported by theoretical and empirical evidence. Empirical models are expressed by algebraic equations for movement of hot particles and heat transfer coefficients [13], [14]. In semi-empirical models, which improve empirical models, more precise ratios for fire propagation velocities expressed by linear differential equations are used [15]. Physical models combine differential equations of thermodynamics with forest fire parameters [16], [17].

Machine learning models use the genetic algorithm to draw on the results of measurements obtained from a number of sensors and cameras placed in different topographical areas. The genetic algorithm is designed to optimize of all types of "submodels" by minimizing the error of general modelling expressed as a percentage of the forest destroyed by the fire and the time when the event occurred.

3. OUR APPROACH

At the moment it is vital to move from solving individual problems to the common methodology for solving them, which was described in our monograph [9].

To explain our methodology, we need to point out that the transition to Industry 4.1 requires a paradigm shift in mathematical modelling. The traditional mathematical modelling of a real object involves two stages. At the first stage the object is described with a differential equation or a system of differential equations (ordinary or partial derivatives) with initial, boundary and another additional conditions. The second stage involves numerical solution to these equations with maximum precision; designing the control system on the basis of the differential model, etc. If further observations of the object conflict with the calculations that have been made, the ongoing processes are studied additionally. The model of the object is refined on the basis of these observations. Then computational studies of this model are conducted again.

Such an approach does not require many intellectual resources, and it is not time-consuming. To make mathematical modelling more efficient we suggest changing the traditional perspective on the differential model of an object. We do not perceive it as accurate background information for further research, but rather as approximate data about an object along with the measured data. On the basis of this information we make a set of mathematical models for the object. The parameters of these models can be changed while the object is in operation. The model which best matches the object at a particular stage of its lifecycle can be chosen from this set.

Our methodology involves three simple steps [9]. The first step is an assessment of the quality of the mathematical model through the functional. In [9] this step was illustrated with a number of sample problems. The second step involves choosing the type of a neural network which is the most SPBPU IDE'20, October 22-23, 2020, Saint - Petersburg, Russia

appropriate for solving the problem. In [9] we described the types of neural networks which we consider to be the most useful. We also offered recommendations on how to choose a certain type of a neural network in accordance with the characteristics of the problem that needs solving. The third step is neural network training, by which we mean minimizing the functional mentioned above and characterizing the quality of the model. The algorithms of such training were considered in the third chapter of our monograph [9]. A major part of the algorithms entails simultaneous adjustment of the neural network parameters as well as its structure. The book contains the results of a considerable number of computational experiments. The examples of building neural network models of the real objects can be found in the articles [18-32].

The procedure of creating a neural network model of a particular element (process) in the object for which we are developing a model will be described here. The element (object) in question is described as a boundary problem for a differential equation:

$$A(u) = g, u = u(x), x \in \mathbb{R}^p, \ B(u)_{\Gamma} = h$$

$$\tag{1}$$

Here u is a function describing the condition of the element or process under consideration, A(u) is a differential, integrodifferential or differential-algebraic operator, i.e. an algebraic expression that contains derivatives, integrals or algebraic relations of function u, B(u), is an operator determined by the boundary conditions, Γ is a boundary of the domain Ω

The approximate solution to the problem (1) will be presented as an output of the artificial neural network (ANN) with the specified architecture:

$$u(x,w) = \sum_{i=1}^{N} c_i v(x,a_i)$$
(2)

Here w is a vector of weights, which aggregates linear input parameters c_i and non-linear input parameters a_i . The basis neural network element (function of v) is set by choosing the type of a neural network and the activation function.

The vector of ANN weights is founded as the result of the process of the stepwise network training, which, in general terms, is based on minimization of a particular error functional. For problem (1) it is made up of two summands, where the first one is the estimation of the equation satisfaction, and the second one - of the boundary condition. Since the model used as the digital twin is supposed to consider the results of observations on the target object, the third summand is added to the functional. It shows how precisely the model (2) conforms to these data.

The computational experiments have illustrated that using a fixed number of test (trial) points does not make sense because in this case minor errors in these trial points will entail errors in other points of area Ω . It appeared that the problem could be solved by using periodically regenerated test points in area Ω , and, if necessary, on its boundary Γ .

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The regeneration of the test points after a certain number of steps of neural network training process will be conducive to its sustainability. In addition to that we organize the calculations as the process of the functional set minimization. Each functional is obtained by a specified selection of the test points and is not totally minimized (only several steps of the selected minimization method are made between the regenerations of the test points). In particular, such an approach allows avoiding the problem of falling into the local extremum. This problem is quite common for most methods of the global non-linear optimization. Furthermore, in the course of error functional optimization, we can include new observations as additional summands.

In addition to that in the monograph [9] the authors offer the methods of designing approximate multi-layer solutions to differential equations on the basis of classical numerical methods. We use well-known formulae of numerical methods to solve differential equations to generate a set of adaptive functional solutions rather than to generate tables of numerical solutions. We have considered a number of problems with real measurements, for which our models represent the object more accurately than exact solutions to the original differential equations.

The specific features of our approach can be illustrated with the following example. In the article [33] the authors define a digital twin for a single-degree-of-freedom dynamic system, the equation of motion of which is described by the following equation:

$$m\frac{d^{2}u(t)}{dt^{2}} + c\frac{du(t)}{dt} + ku(t) = f(t)$$
(3)

Here m, c and k are the mass, damping and stiffness coefficients. The forcing function and the dynamic response is denoted by f(t) and u(t) respectively.

If the coefficients are fixed, the problem has an analytical solution. In the course of finding the solution, the authors introduce the values which can be measured with sensors, suggesting that the model describes an aircraft.

If the coefficients are fixed, the problem has an analytical solution. In the course of finding the solution, the authors introduce the values which can be measured with sensors, suggesting that the model describes an aircraft. Such values $\frac{k}{k}$

can be undamped natural frequency $\omega = \sqrt{\frac{k}{m}}$ and damping

factor
$$\zeta = \frac{c}{2\sqrt{km}}$$
.

The model corresponding to the equation (3) a bi-time-scale model reproducing the dynamics of the physical system at both time-scales, the system time and a "slow time", is defined as a digital twin. The concept of a slow time is used to separate the evolution of the system properties from the instantaneous time. The "slow" time corresponds to the data about the condition of the measured system parameters that are received from the sensors. The article only considers the situations when the mass and stiffness coefficients change separately or together. It also addresses the issue of error in measuring values ω and ζ and its influence on the result. In addition, the authors of the article do not suggest explicitly using the data about the object that cannot be easily inserted in the differential equation. However, in our neural network approach such data are certainly taken into consideration by applying the common error functional.

In particular, the equation (3) can apparently be regarded as quite an approximate model of an aircraft. We solved the problems in which the differential equation was considered to be one part of the information about the behavior of the real object, whereas the results of measurements were the other part. Moreover, our approach allows solving the problems of building the models on the basis of the equation (3) without any information available about the slowly changing coefficients in the situation when they are restored while the object is operating in accordance with the measurements of the value u(t).

5. CONCLUSION

We recommend using classical methods to create digital twins of the technical objects, in which the processes are accurately described with differential equations. If the accurate differential models of the technical objects are unknown, we recommend using neural network modelling to create their digital twins [9].

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