A Fast PSO Algorithm Based on Alpha-stable Mutation and Its Application in Aerodynamic Optimization

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***Abstract*—**In order to balance the global and local search ability of the basic particle swarm optimization (PSO) in the evolution loop, an alpha-stable distribution is adopted and applied to perform mutate operation in PSO. The development of a new dynamic mutation particle swarm optimization algorithm was established by the alpha-stable mutation. By dynamically modifying the stability coefficient α of alpha-stable function, the amplitude and intension of the mutate operation is adjusted adaptively, and the global optimization ability of PSO is improved. The new algorithm is compared with DE and PSO on seven test functions . Simulation results show that the alpha-stable PSO algorithm have a faster convergence speed and a better global optimization performance in low, medium and high dimension problems. The proposed algorithm is applied to drag reduction design of RAE2822 transonic airfoil and compared with PSO algorithm. The comparison results also show that our algorithm is more excellent than basic PSO.

Keywords-Particle Swarm Optimization; alpha-stable distribution; dynamic mutation ; aerodynamic optimization

I. INTRODUCTION

The development of aviation science and technology has made a higher requirement for aerodynamic optimization design of aircraft. By means of various optimization algorithms, the design and analysis of the airfoil and wing body assembly, as well as the whole machine reduces drag and noise can be realized. It provides support for the development of various new aircraft.

Optimization algorithm is one of the key technologies of aerodynamic optimization design [1]. According to the difference of optimization mechanism and optimization behavior, optimization algorithm can be divided into two categories: Classical optimization algorithm and intelligent optimization algorithms. The intelligent algorithm represented by the Particle Swarm Optimization (PSO) [2] and the Deferential Evolution (DE)[3] has gained more attention due to its good parallel efficiency, global and robustness. Because of its simple algorithm, low parameter and strong optimization ability, PSO algorithm is one of the most widely used algorithms in aerodynamic optimization. However, it still has inherent defects. In a later stage of the evolution, the algorithm focuses on the exploration, which makes it easily fall into the local extreme point. And it also has a slow convergence speed, as well as poor accuracy. Therefore, the researchers carried out a series of research work and proposed several improved PSO algorithms successively. Shi et al [4] proposed the standard PSO algorithm. This method increased the inertial weight of the velocity update equation to keep the example moving inertia, which made it expand the search space trend. However, because the inertia weight of the PSO algorithm of fixed parameters is usually less than 1, it is easy to show premature convergence due to the smaller particle velocity. The KPSO [5] uses the compression factor to modify the speed update equation to control system behavior convergence. Although the method can get a higher quality solution, the algorithm also has the disadvantage of falling into local optimum. If the inertia of the original algorithm was replaced by the second order oscillation, it could have increased the diversity of the population to form an improved second-order oscillating particle swarm algorithm (SOPSO)[6]. By adjusting the parameters, the algorithm can improve the early particle acquisition ability and the accelerated convergence performance. However, the process is more complicated and requires several attempts. Sun et al[7]. proposed the quantum particle swarm optimization algorithm(QPSO). The method uses wave function to describe the motion state of the particle, which is a global convergence algorithm. All the same, the algorithm is prone to lack of population diversity, and thus falls into local optimal solution. Furthermore, the hybrid algorithm with other algorithms is one of the improvement directions. Lei[8] has developed a new hybrid PSO algorithm, GAPSO, with the characteristics of biological evolution, which has high convergence speed and can increase population diversity, and improve population evolution quality. The algorithm introduces the speed variation operation and position cross operation, which has a better chance to escape the local extreme point and improve the convergence speed and global convergence. Yet, there are a lot of human factors that make this method weak. In conclusion, regardless of the improved PSO algorithm, the starting point is how to improve the global convergence while improving the convergence speed of the algorithm, and avoid getting into local optimization. However, the method and application ability of these algorithms have advantages and disadvantages.

In this paper, the shortcomings of PSO algorithm widely used in aerodynamic optimization design of aircraft are studied. Based on the comprehensive analysis of various improved algorithms, a new particle swarm optimization algorithm based on alpha-stable dynamic variation is proposed. The optimization ability of low, medium and high dimensional design variables is analyzed by using multiple functions. Then, it is applied to the field of aerodynamic optimization, which is used to analyze the design of drag reduction of two-dimensional airfoil (RAE2822).

II. OPTIMIZATION ALGORITHM

A. PSO Algorithm

The Particle Swarm Optimization algorithm (PSO) was originally proposed to simulate the flight foraging behavior of birds. It was initialized to a group of random particles and finds the optimal solution through iteration. In each iteration, the particle updates itself by tracking two extreme values. The first extremity is the optimal solution found by the particle itself, known as the individual extreme. The other extreme is the optimal solution that the entire population currently finds which is called the global extreme. When the above two optimal values are found, the particle can update its own speed and new position according to formula 2.1.

$$v_{id}^{k+1} = v_{id}^{k} + c_1 r_1 (pbest_{id}^{k} - x_{id}^{k}) + c_2 r_2 (gbest_{id}^{k} - x_{id}^{k}) \tag{2.1}$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1} \tag{2.2}$$

In formula 2.1, c_1 and c_2 are accelerated constants, and the range of values is [0,4]. The general values are: $c_1 = c_2 = 2$. r_1 and r_2 are two random variables that are uniformly distributed within [0,1]. The positions and velocities of each particle are initialized in a random manner, and then the particles move in the direction of the global optimal and individual optimal direction.

B. Analysis of the alpha-stable distribution

Alpha-stable distribution, also called fractal distribution, which was originally established by Levy et al.[9-10], is a kind of widely used stochastic signal model. Its distribution is represented by scale factor, characteristic index, displacement parameter and skewness parameter. And the probability density function can be defined by the continuous Fourier transform of the characteristic function.

$$f(x;\alpha,\beta,\sigma,\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp[it\mu - |\sigma t|^{\alpha}(1 - i\beta\,\mathrm{sgn}(t))\Phi] e^{-itx} dt \tag{2.3}$$

The sgn(t) is the symbol of t. The Φ is expressed as below:

$$\Phi = \begin{cases} \tan(\pi\alpha/2), \alpha \neq 1 \\ -(2/\pi)\log|t|, \alpha = 1 \end{cases} \tag{2.4}$$

The characteristic function of a stable distribution can be determined by four parameter $(\alpha,\beta,\gamma,\sigma)$, $\alpha \in (0,2]$ is the stability parameter; $\beta \in [-1,1]$ is the skewness parameter. When $\beta = 0$, it represents a stable distribution of symmetric Alpha or SaS; $\gamma \in [0,+\infty)$ is the scale parameter; and $\sigma \in (-\infty,+\infty)$ is the location parameter. When α=2, it represents Gaussian distribution. The mean value is σ, and the variance is $\gamma\sqrt{2}$. When $\alpha=1, \beta=0$, it represents Cauchy distribution.

Fig. 1 shows the probability density function curve of standard SaS distribution under different α. When $\alpha=2$, the probability density function curve of SaS is consistent with the Gaussian distribution. When $\alpha=1$, it's consistent with Cauchy distribution. And it still retains the characteristics of many Gaussian distributions: smooth, unimodal distribution, and median or mean symmetry. Comparing the normal distribution probability density function with the SaS density function, we can understand the following differences. When the absolute value of x is relatively small, the peak of the normal distribution of the SaS density function is more pointed. For some intermediate values, the SaS distribution is lower than normal distribution. Most importantly, the probability density function of the SaS distribution has a thicker trailing tail than the Gaussian.

C. PSO-Alpha_stable mutation

In the alpha stable distribution, the stability coefficient α is a very important parameter. It describes the size of the tail of the distribution and determines the range of random Numbers generated. In this paper, the Alpha-stable distribution is used to generate random Numbers and to perform mutation operation on individuals in PSO population. In the process of variation, the range and amplitude of variation are realized by dynamically adjusting the stability coefficient α of alpha-stable function. Its variation range is [1,2], and its change process is shown in Fig. 2. At the beginning of the PSO cycle, the value of α is small. At this time, it has strong mutation ability, which can help to enhance the global search ability of the algorithm, so as to avoid premature local optimization of the algorithm. As the A value gets larger and larger, the global mutation ability decreases, but local search capability increases. Eventually, α is going to increase to 2. At this time, the traditional Gaussian variation is carried out. And the local search capability is the strongest, which is conducive to improving the accuracy of the solution. This dynamic Alpha-stable mutation can improve the development ability and exploration ability of the algorithm. The flow chart of the proposed algorithm is shown in Fig. 3

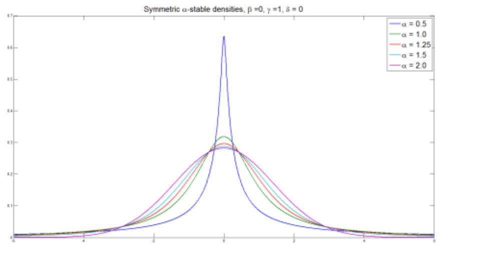

Figure 1 Symmetric α-stable densities under different α

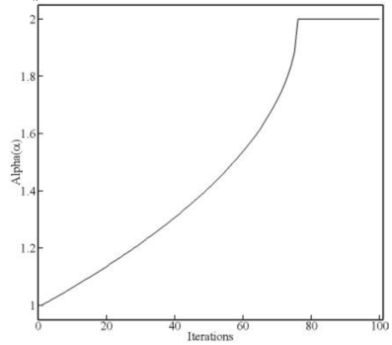

Figure 2 Changing processes of stability coefficient α

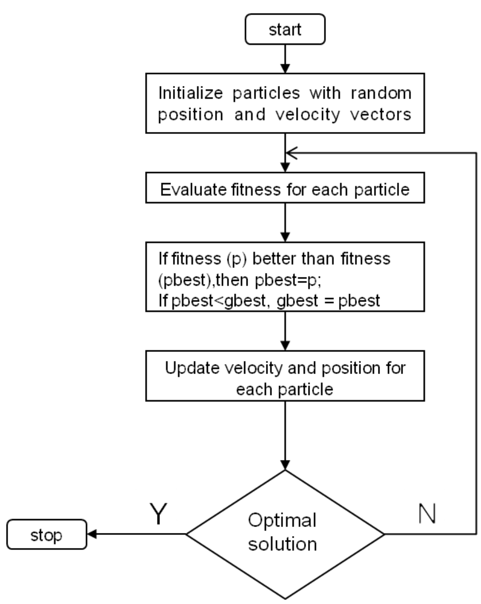

Figure 3 The flow chart of the PSO alpha-stable algorithm

III. FUNCTION TEST AND SIMULATION ANALYSIS

A. *Test functions*

In order to verify the performance of the improved algorithm, the DE algorithm, the basic PSO algorithm and the PSO alpha-stable algorithm are selected for the comparison test. By introducing the problem of seven functions commonly used in PSO and DE algorithm, the numerical analysis is carried out. Table 1 shows the seven functions and their specific attributes for the test. Among them, Sphere function, Schwefel function and Elliptic function are continuous single-peak functions. The Sphere function is a simple square function with a minimum point $f(0,0,0)=0$. It is mainly used to test the accuracy of the algorithm. The function Schwefel and Elliptic are the deformation of Sphere functions, which increase the interaction of each dimension function. And the other four functions are multimodal functions. Although the Rosenbrock function has only one global minimum point $f(1,1)=0$, it is difficult to obtain the global minimum because of the small gradient around the extreme point. Function Ackley, Rastrigin and Griewank are typical nonlinear multimodal functions. They have a large number of local minimum points which are considered to be complex multimodal problems which are difficult to deal with, so they can be used to test the global search performance of the algorithm.

B. *Low dimensional optimization problem searching ability test.*

Firstly, the optimization ability of the three algorithms were compared and tested. The design variables are 30 dimensions, and the population size of the three algorithms is 100. The inertial weight of the PSO algorithm decreases linearly with algebraic linearity in [0.4, 0.9]. The acceleration constant of all algorithms is: c1=c2=1.5. The three algorithms were used to search 30 times respectively, with 1000 times of each iteration. The average objective function value and the minimum target function value are calculated.

Fig. 4 shows the global optimal value convergence curve of the three algorithms under the low dimensional optimization problem. It can be seen that the PSO alpha-stable algorithm can maintain a faster convergence rate than the basic PSO algorithm and the DE algorithm. The PSO alpha-stable algorithm can find the global optimal solution better than the other two algorithms, whether it is a unimodal complex function or a multimodal complex function. Table 2 shows the analysis results of three functions on the low dimensional optimization problem. It can be seen from the average value and variance that the PSO alpha-stable algorithm is better than the other two algorithms. For the 30 - dimensional Rastrigin function, the algorithm can even search the optimal solution directly under the premise of a large number of local advantages of a large number of sinusoidal turning points. Based on the simulation of low dimensional optimization problem, PSO alpha-stable algorithm has better ability than other two algorithms.

C. *The test of optimization ability of medium and high dimensional optimization.*

Extend the design variables to 60 and 100 dimensions respectively in order to test the optimization ability of the three algorithms in the medium and high dimensional problems. The three algorithms were used to search 30 times respectively, with 1000 times per iteration. The average objective function value and the minimum target function value are calculated.

The global convergence curve of the 60-dimensional design variable optimization problem is shown in Fig. 5. The average value and variance results are shown in Table 3. The convergence curve of the 100 - dimensional problem is shown in Fig. 6. The statistical results of the average value and variance of the seven functions are shown in Table 4. With a increased dimension, the optimization accuracy of the three algorithms decreased. Especially in the high - dimensional optimization problem, the accuracy of optimization has been greatly reduced. However, in the case of the same number of calls to the target function, PSO alpha-stable optimization reflects a higher accuracy and better convergence effect on average value and standard deviation. This indicates that it has good adaptability to medium and high dimensional problems.

In conclusion, all of low, medium and high dimension problems, compared with the basic PSO algorithm and DE algorithm, PSO alpha-stable algorithm can obtain the optimal solution of unimodal and multimodal with better convergence speed, stability and robustness. It is a more practical and effective global optimization algorithm.

IV. AIRFOIL DESIGN OPTIMIZATION

A. *Airfoil optimization model*

The aerodynamic shape optimization design was carried out with the minimum drag of the RAE2822 airfoil in the state of $Ma = 0.73, \alpha = 2.79, \mathrm{Re} = 6.5 \times 10^6$ [11]. The constraint condition is Lift coefficient is greater than that of the original airfoil. Meanwhile, the maximum thickness of the airfoil is not reduced, and the pitching moment coefficient is not greater than that of the original airfoil. Its mathematical model is:

$$\begin{cases} Min \ \ f_1(\boldsymbol{X}) = Cd \\ S.t. \ \ g_1(\boldsymbol{X}) = Cl \geq Cl^*, \ \ g_2(\boldsymbol{X}) = c_{\max} \geq c^*, \\ \qquad g_3(\boldsymbol{X}) = Cm \leq Cm^* \end{cases}$$

$\boldsymbol{X}$ is the design variable vector; Cd is the drag coefficient; Cm is the moment coefficient; Cl is the lift coefficient; $c_{\max}$ is the Maximum thickness of airfoil. The superscript $*$ represents the calculated value of the initial airfoil.

B. *Parameterization of the airfoil.*

The parameterization of the airfoil was designed to accurately describe the original model with as few parameters as possible and to provide as many design schemes as possible. The parameterization method used in this paper is the CST method using the six-step Bernstein polynomial function. Six design parameters are taken from the upper and lower surfaces of the airfoil. There are 12 optimization design variables.

By using the CST parameterized method to fit the original airfoil, the maximum fitting error of the upper wing is 0.0002, and the maximum fitting error of the lower plane is 0.0005, which can meet the engineering application requirements. Fig. 7 shows the comparison of surface pressure coefficients of the parameterized wing with the original airfoil. And you can see that they're consistent. This indicates that the six-step Bernstein polynomial CST method used in this paper can accurately describe the design space of the RAE2822 airfoil.

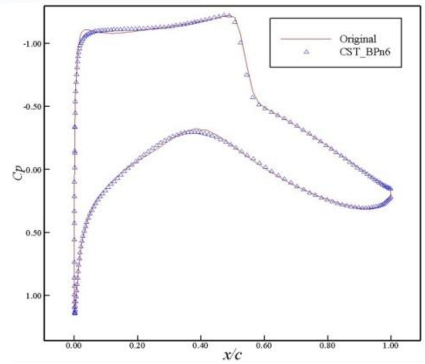

Figure 7 comparison of pressure coefficients of RAE2822 airfoil CST parameters.

C. *Optimization results and comparison*

PSO-alpha stable algorithm and PSO algorithm were used to optimize the transonic single point optimization design and compare the effects of the two in optimization. The population size of both is 25 and the initial population size is the same, the evolutionary algebra is 39, and the inertial weight factors *A* decreases to 0.4 with the iterative evolution from 0.9 linear differential. $c_1=c_2=0.5$.

Fig. 8 shows the time course of optimization of drag coefficient of two algorithms. Figure 9-10 shows the geometric shape and pressure distribution of the airfoil before and after optimization of the two algorithms. Table 5 shows the comparison of the geometrical characteristics of the optimized rear airfoil with the original airfoil. It is shown that comparing with the original airfoil, the two algorithms have achieved the goal of reducing resistance on the basis of satisfying the constraint conditions. However, Fig. 8 shows that as the number of evolutionary algebra increases, the convergence rate of PSO algorithm is slower. And the PSO alpha-stable algorithm still has a faster convergence performance and a better global optimization. Fig. 9 demonstrates that the maximum thickness position of the optimized airfoil is moved backward. And the upper surface vertex is moved at the same time, so that the upper flange of the optimized airfoil is fuller. Fig. 11 is the pressure cloud diagram, which shows the increase of the peak suction peak of the airfoil. And the torque characteristics can be maintained due to the increase of the loading at the same time. The shock wave of the original airfoil was significantly eliminated and the pressure distribution of the upper wing was smoother. As a result, the optimized RAE2822 airfoil wave resistance significantly decreases, thus reducing the resistance of the whole airfoil, and the lifting resistance ratio also has a more obvious improvement. RAE2822 airfoil aerodynamic optimization design of transonic slowdown numerical example shows that compared with the basic PSO algorithm, the PSO alpha-stable algorithm in the complex optimization problems have a batter adaptability. Under the same population size and evolution algebra, it has a faster convergence speed and a stronger global optimization ability. The effect of optimization is more ideal.

V. CONCLUSIONS

In this paper, a new particle swarm algorithm was built and the reference verification was made by combining the alpha-stable variation theory. The conclusions are as following :

1). As a new swarm intelligence algorithm, PSO algorithm has been widely used in many practical engineering fields. However, in a later stage of the evolution, with the disappearance of population diversity, the basic particle swarm algorithm was prone to precocious stagnation and trapped in the local optimal phenomenon. Therefore, corresponding improvement is needed.

2). For the defects of PSO algorithm, the individual in PSO population was mutated by using the alpha-stable distribution in PSO algorithm. And by dynamically adjusting the stability coefficient α of alpha-stable function, the global optimization ability was improved. It can be known from the test function that the new algorithm can show better searching ability and global search performance whether it is in low, medium or high dimension.

3). The new PSO alpha-stable algorithm could obtain better airfoil optimization results than the basic PSO algorithm in the same evolutionary algebra and iteration times in the design of the RAE2822 transonic drag reduction optimization. This proves that the algorithm is valid.

Although in this article PSO-alpha stable algorithm shows better results in test function and airfoil optimization, but for a more general aerodynamic optimization design of complex function and three-dimensional configuration application effect are still to be further tested.

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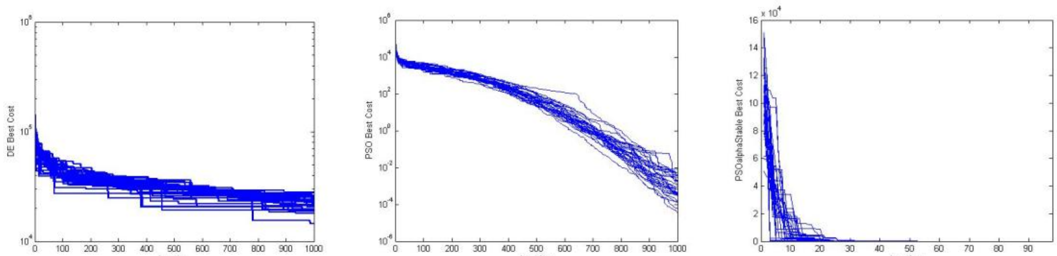
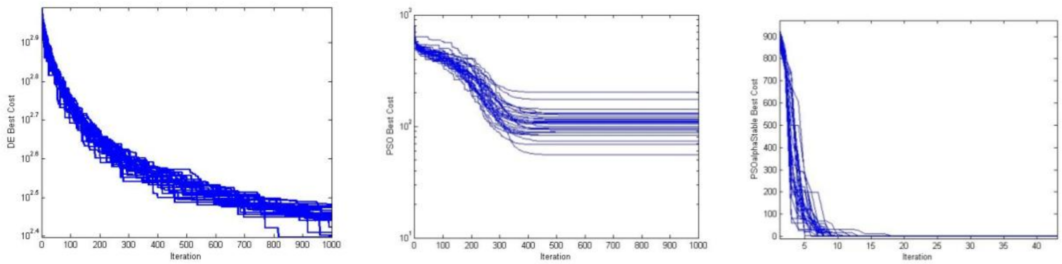


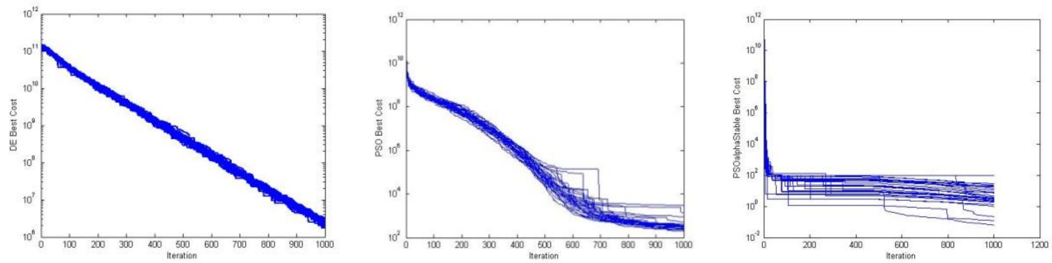
a) DE b) PSO c) PSOalphastable

Figure 4 The global convergence of the three algorithms for low dimensional optimization



a) DE b) PSO c) PSOalphastable

Figure 5. The global convergence of the three algorithms for medium dimensional optimization.



a) DE b) PSO c) PSOalphastable

Figure 6. The global convergence of the three algorithms for high dimensional optimization.

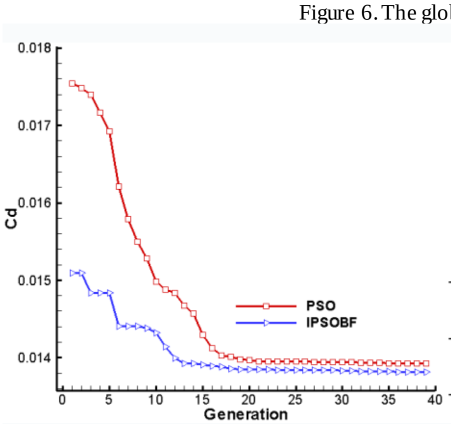


Figure 8 Progress of airfoil optimization

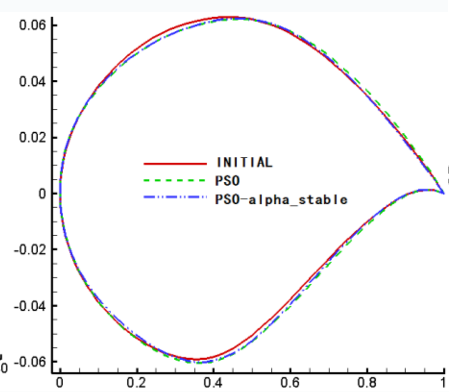


Figure 9 Comparison of airfoil shape

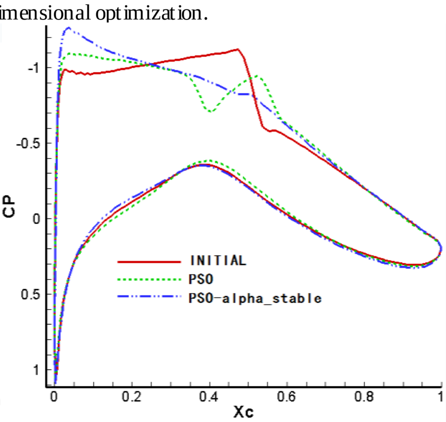
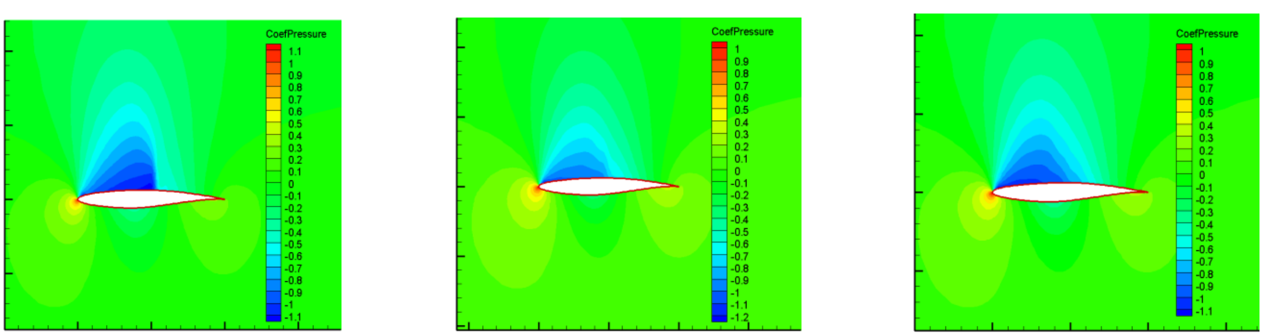


Figure 10 Comparison of pressure distribution



a)Initial b)PSO c)PSO-alpha_stable

Figure 11 Comparison of pressure contour distribution

TABLE 1. TEST FUNCTION

Test function	name	search space	attribute	optimal solution
$f_1(x)=\sum_{i=1}^{n} x_i^2$	Sphere	[-100,100]	unimodal	0
$f_2(x)=\sum_{i=1}^{n}(\sum_{j=1}^{i} x_j)^2$	Schwefel	[-100,100]	unimodal	0
$f_3(x)=\sum_{i=1}^{n}(10^6)^{\frac{i-1}{n-1}} x_i^2$	Elliptic	[-100,100]	unimodal	0
$f_4(x)=\sum_{i=1}^{n} 100(x_{i+1}-x_i^2)^2+(1-x_i)^2$	Rosenbrock	[-100,100]	multimodal	0
$f_5(x)=-20\exp(-0.2\times\sqrt{(\frac{1}{30}\sum_{i=1}^{n} x_i^2)}) - \exp(\frac{1}{30}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	Ackley	[-32,32]	multimodal	0
$f_6(x)=\sum_{i=1}^{n}(x_i^2-10\cos(2\pi x_i)+10)$	Rastrigin	[-5.12,5.12]	multimodal	0
$f_7(x)=\frac{1}{4000}\sum_{i=1}^{n} x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	Griewank	[-600,600]	multimodal	0

TABLE 2. COMPARISON OF CALCULATION RESULTS OF LOW DIMENSIONAL OPTIMIZATION PROBLEMS.

Function / algorithm		f1	f2	f3	f4	f5	f6	f7
DE	average value	7.79E-12	2288.15	1.27E-08	65.7313	6.97E-07	59.7265	2.06E-10
	standard deviation	2.91E-12	3100.6392	4.11E-09	22.9936	1.21E-07	4.7132	4.45E-10
PSO	average value	1.52E-40	1.5645 E-3	2.93E-37	30.0815	0.031043	38.6044	0.01165
	standard deviation	4.11E-40	1.99 E-3	6.31E-37	24.8825	0.17003	10.5568	0.012338

PSO-alpha_stable	average value	0	0	0	0.032194	7.40E-15	0	6.34E-09
	standard deviation	0	0	0	0.042649	1.35E-15	0	6.43E-09

TABLE 3. COMPARISON OF CALCULATION RESULTS OF MEDIUM DIMENSIONAL OPTIMIZATION PROBLEMS.

Function / algorithm		f1	f2	f3	f4	f5	f6	f7
DE	average value	4.27E-03	132544.0	3.8647	1442.3691	0.013124	282.983	6.01E-03
	standard deviation	6.58E-03	8833.384	0.59278	253.9378	0.00129	13.2058	1.88E-03
PSO	average value	2.12E-13	385.7837	2.02E-09	82.3455	1.4562	107.6875	1.19E-02
	standard deviation	6.91E-13	263.3214	7.01E-09	39.0484	0.60274	30.0375	2.0073
PSO-alpha_stable	average value	8.97E-08	0.036105	0	10.6555	9.98E-06	2.06E-06	1.24E-05
	standard deviation	4.91E-17	0.078582	0	20.6875	5.47E-05	8.43E-06	1.03E-05

TABLE 4. COMPARISON OF CALCULATION RESULTS OF HIGH DIMENSIONAL OPTIMIZATION PROBLEMS.

Function / algorithm		f1	f2	f3	f4	f5	f6	f7
DE	average value	32.0608	368656.08	18683.689	2436751.7	2.3588	691.2031	1.2754
	standard deviation	2.9631	30912.335	1925.8923	432694.94	0.1031	20.4637	0.033412
PSO	average value	0.015059	7254.0259	202.4868	472.2732	2.5875	180.0558	0.092413
	standard deviation	0.042626	2231.435	760.3729	535.4626	0.52953	29.7961	0.11871
PSO-alpha_stable	average value	1.40E-05	7.5659	4.29E-04	13.5549	1.52E-04	1.53E-04	8.28E-03
	standard deviation	7.02E-05	14.9846	2.02E-03	23.7622	5.44E-04	2.52E-04	0.033773

TABLE 5. COMPARISON OF AERODYNAMIC PERFORMANCE

	Initial	PSO	PSO-alpha_stable
Cl	0.74	0.74	0.74
Cd	0.0121	0.009737	0.009693
Cm	-0.089	-0.07647	-0.08069
c_{max}	0.1211	0.1227	0.1211